25 YEARS AFTER THE DISCOVERY: SOME CURRENT TOPICS ON LENSED QSOs

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Analogy Between Gravitational and Optical Lenses

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“ANALOGY BETWEEN GRAVITATIONAL AND OPTICAL LENSES”

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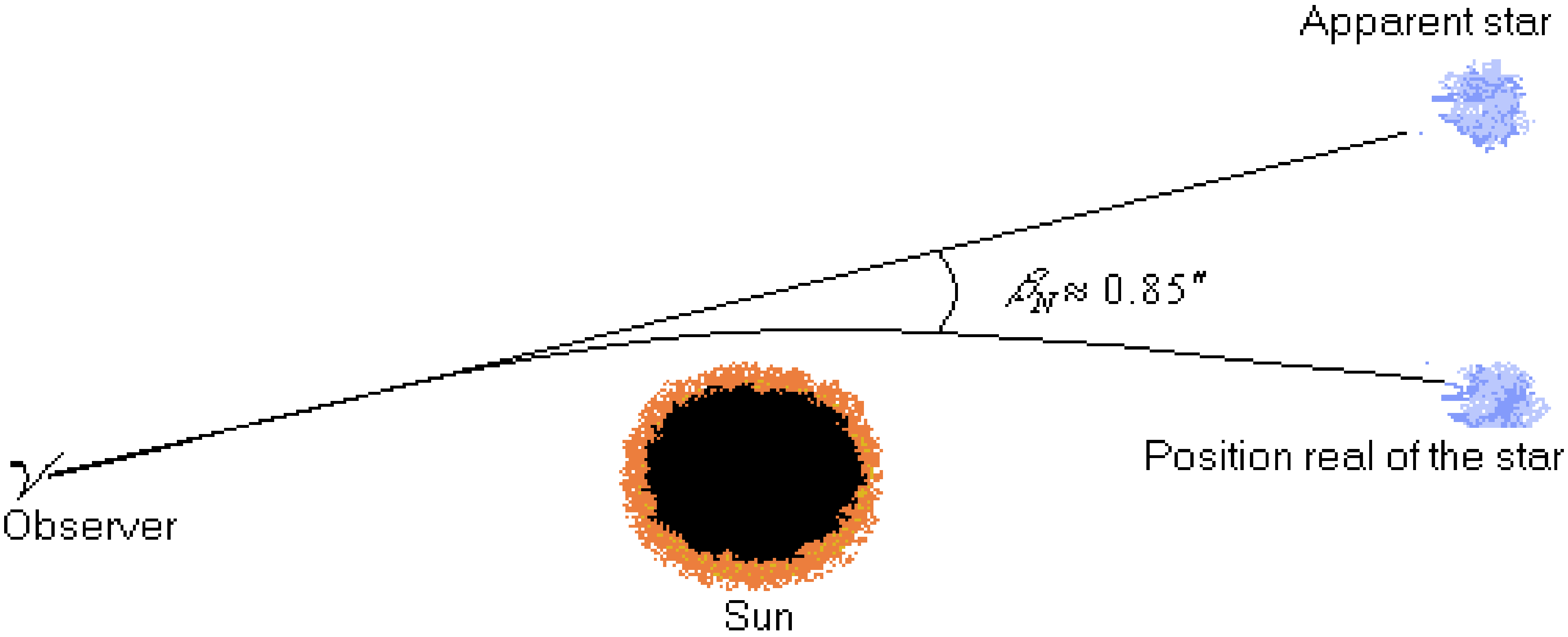
In this paper alternatives calculus are proposed intended to simplify the getting of equations given by gravitational lenses characteristics. To do it, an analogy has been done between proposed calculations by classic mechanics and the theory of general relativity with those of optical lenses. This analogy begins from those systems where the deflector, observer and the source are aligned; and when there is not a perfect alignment between these. From this situation the necessary conditions are deduced for multiple images formation according to the masses distribution in the gravitational lens.

In 1804, Soldner was the first who calculated that, for small angles, the Newtonian deflection of light by a massive object should be :

$$1 \longrightarrow \beta_N = \frac{2\,GM}{C^2 R}$$

Where M is the mass of the deflecting object and R is the deflection impact parameter.

For a light ray grazing the sun this gives a deflection angle $\beta \approx 0.85''$



Newtonian angle of the deflection of light by the Sun

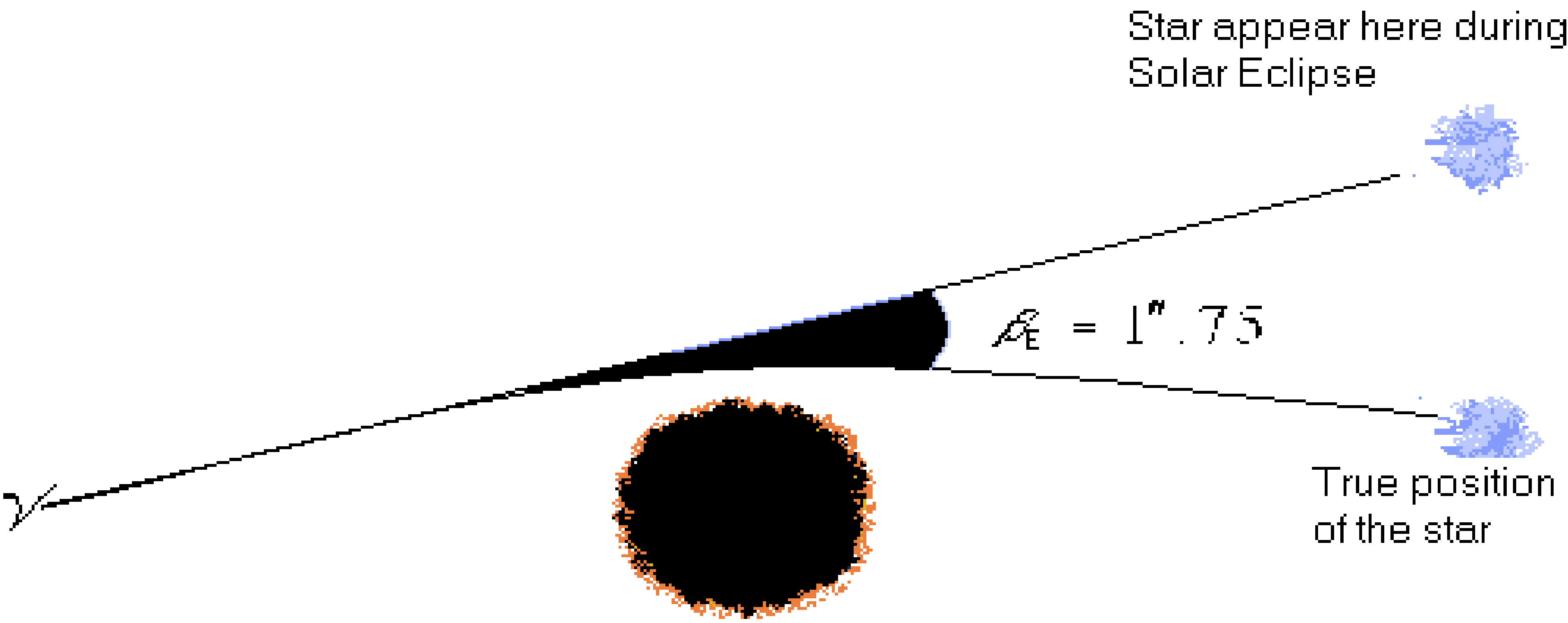
Figure 1

General Relativity predicts that the bending angle for a light ray in the vecinity of a point mass to be :

$$2 \longrightarrow \quad \beta_E = \frac{4\,GM}{c^2 R}$$

precisely double the value expected from Newtonian gravity. With this, ligth rays grazing the surface of the Sun are bent by an angle of $1''.75$

This value is not twice that obtained by Soldner due to differing estimates for the solar mass an radius at the time of calculation.



Einstein´s angle of deflection of the light by the Sun. Eddington´s expedition.

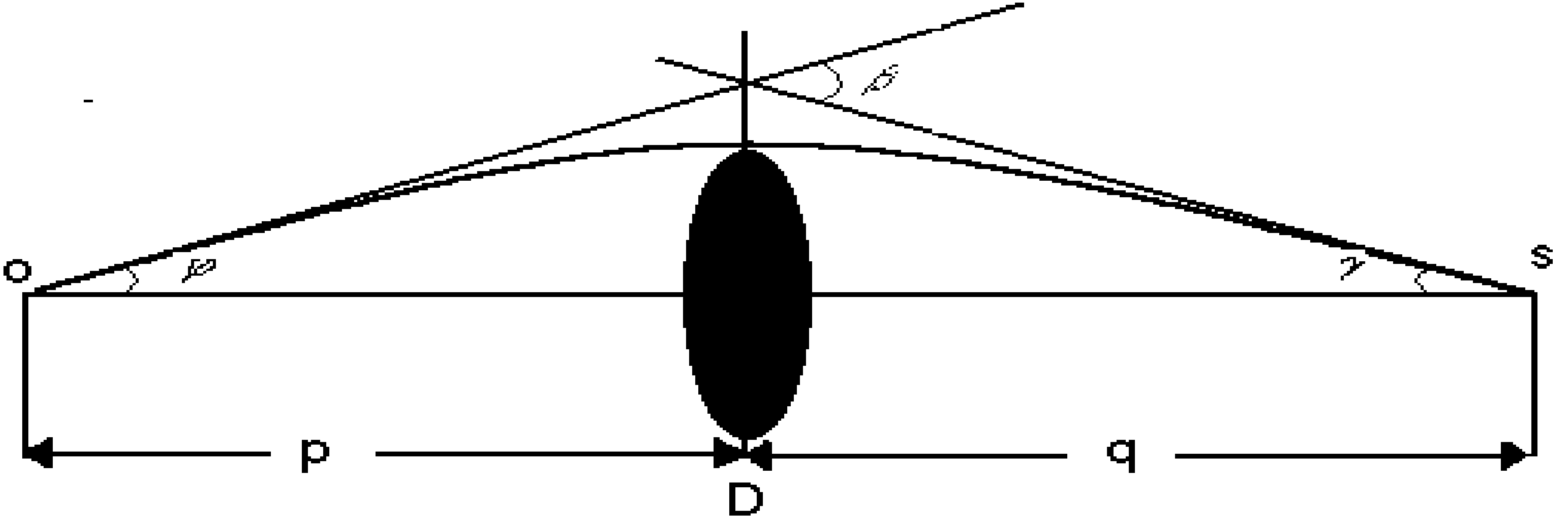
Figure 2

In the following decades, light deflection or **Gravitational Lensing** (GL) was only rarely the topic of a research paper: In 1924, Chwolson mentioned the idea of a fictitous double star and the mirror reversed nature of the secondary image. He also mentioned the symmetric case of star exactly behind star, resulting in a circular image. Einstein also reported in 1936 about the appearance of a luminous circle (Einstein Ring) for perfect alignment between source and lens, and of two magnified images for slightly diplaced positions. Influenced by Eisntein, Fritz Zwick pointed out in 1937 that galaxies (extragalactic nebulae) are much more likely to be lensed than stars amd that one can use the gravitational lens effect as a natural telescope.

In the 1960´s, a few partly independent theorical studies showed the uselfulness of lensing for astronomy. In particular, Sjur Refsdal derived the basic equations of Gravitational Lens Theory and subsequently showed how the gravitational lens effect can be used to determine Hubble´s Constant by measuring the time delay between two lensed images.

ANALOGY BETWEEN GRAVITATIONAL AND OPTICAL LENSES

when calculate the characteristic parameters of the Gravitational Lenses with the equation for relativistic deviaton angle and take into account graph geometry, we can to solve for:



1. **Deflector, Observer and the Source are Aligned**

Figure 3

From simple geometry , we can obtain **p** and **q**, where p and q are the angular diameter distances between lens and source plane and observer and source plane, respectively (see Figure 3) ρ is the impact parameter, 2φ is the angle with s see to lens **D**

$$3 \longrightarrow \qquad p = \frac{\rho^2(c^2 q + 4MG)}{4MGq - c^2\rho}$$

$$4 \longrightarrow \qquad q = \frac{\rho^2(c^2 p + 4MG)}{4MGp - c^2\rho}$$

With the equation **3** or **4** we can obtain the equation of the lens for a ray from come to source, deflector, and observer are aligned, apart from the source is located in a finite distance.

$$\mathbf{5}\ \longrightarrow\ \left(\frac{1}{q}+\frac{1}{p}\right)=\frac{4MG}{c^{2}}\left(\frac{1}{\rho^{2}}-\frac{1}{pq}\right)$$

The distances q and p are common more than ρ ,therefore we can despise $\frac{1}{pq}$. We can put forward the focal distance equation of the graviational lenses :

$$\mathbf{6}\ \longrightarrow\ F=\frac{\rho^{2}c^{2}}{4MG}$$

This equation is exactly equal to expression for optic thin lens:

$$\mathbf{7}\ \longrightarrow\ \frac{1}{F}=\frac{1}{q}+\frac{1}{p}$$

2. Is not a perfect alignment between Source, Observer and Deflector

We study more general equation for a emit ray light by source s situate a finite distance from deflector D on the optic axis to distance h.

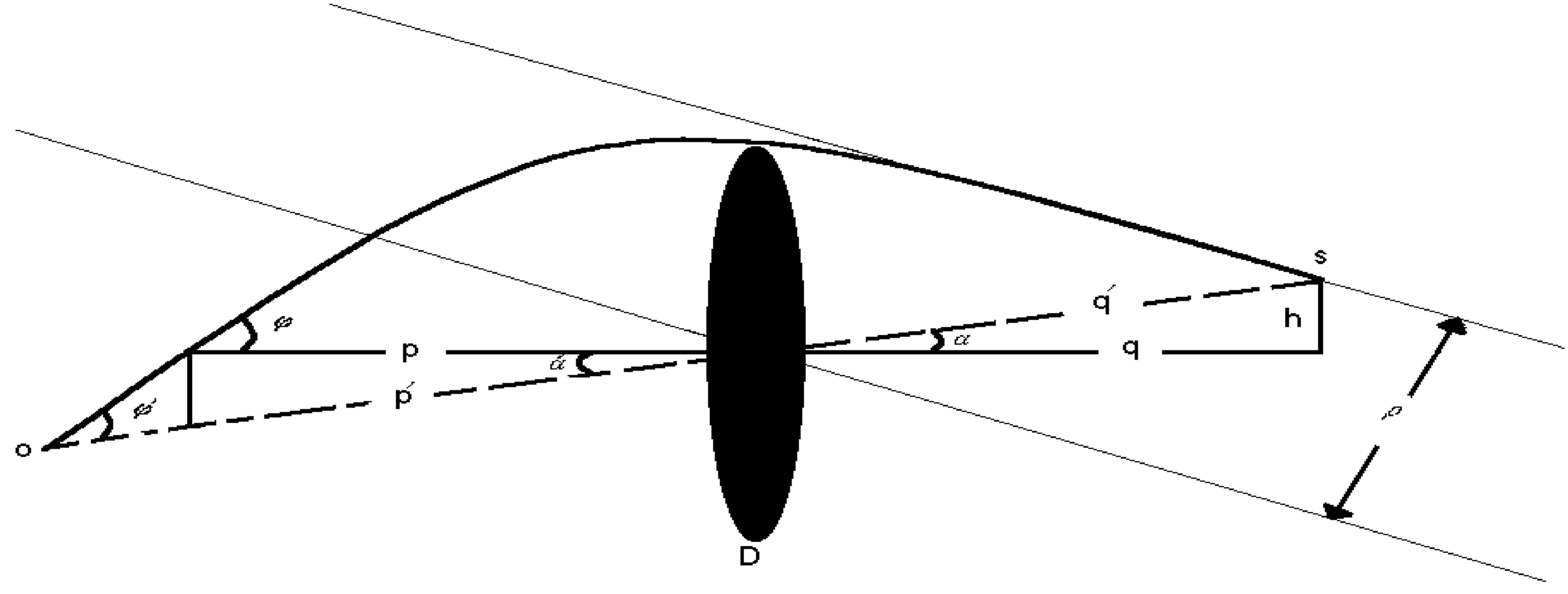


Figure 4

We can write a equation for the parameters p' and q' (see Figure 4)

$$8 \longrightarrow \frac{1}{q'} + \frac{1}{p'} = \frac{1}{F} - \frac{\rho^{2}}{F\,p'q'}$$

It is easy to obtain the distances relation

9 $\longrightarrow$ $p' = \frac{p \sin(\varphi + \alpha)}{\sin \varphi}$ **10** $\longrightarrow$ $q' = Sec\ \alpha$

If we introduce p' and q' in equation 8, we find:

11 $\longrightarrow$ $\frac{\cos \alpha}{q} + \frac{\sin \varphi}{p \sin(\alpha + \varphi)} = \frac{1}{F} - \frac{\rho^2 \cos \alpha \sin \varphi}{Fpq \sin(\alpha + \varphi)}$

Where $\alpha = \arcsin h/q'$, h is the distance of the source s over optic axis and q' is the distance from the source s to the deflector D.

This is the general form equation $\frac{\cos \alpha}{q} + \frac{\sin \varphi}{p \sin(\alpha + \varphi)} = \frac{1}{F} - \frac{\rho^2 \cos \alpha \sin \varphi}{Fpq \sin(\alpha + \varphi)}$ for the analysed case.

if we take $\alpha = 0$,find this equation $\left(\frac{1}{q} + \frac{1}{p} \right) = \frac{4MG}{c^2} \left(\frac{1}{\rho^2} - \frac{1}{pq} \right)$, with the mathematical condition h=0.

Multiple Images

Is important to know, there are a spherical distribution mass, and this produce the formation of multiple images.
We can show, there are two rays emit by a source s (see Figure 5)
to arrived observer o, and therefore produce the multiple images formation of the source s.

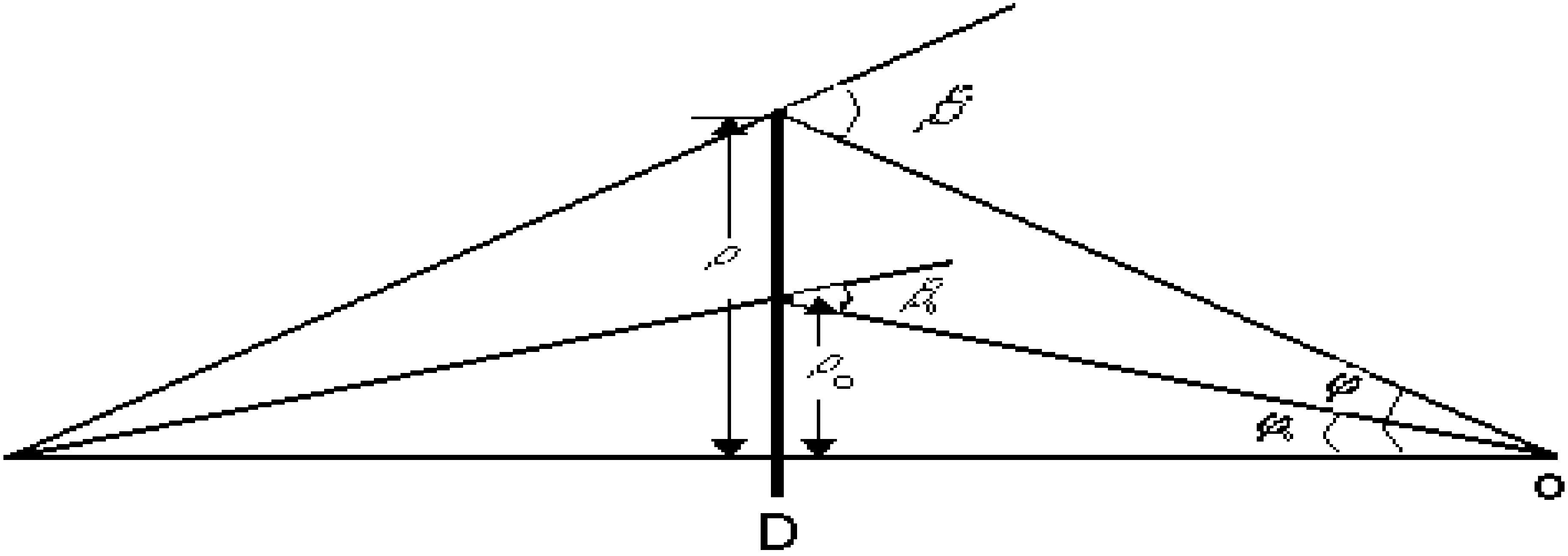


Figure 5

For a deviation angle β , the condition for the other image formation is:

$$12 \longrightarrow \beta \geq \beta_0$$

For the observer can to see the source, be fullfilled the follow condition:

$$\frac{\varphi_0}{q} \approx \frac{\beta_0}{q+p}$$

Expressing the angle φ_0 between the direct ray and the incoming deflected ray as: $\varphi_o = \frac{R}{P}$,and for the angle φ is $\varphi = \frac{\rho}{p}$

We do the follow aproximation: $\beta_0 = \frac{R}{pq}(p+q) \approx \frac{\rho}{pq}(p+q)$

It is easy to obtain the expression : $\frac{M(\rho)}{\pi\rho^2} \geq \frac{c^2}{4\pi G}\frac{q+p}{pq}$

Where the left side of the before expression, is called the apparent surface mass density:

$$\Sigma\rho$$

The right side, value constant is called the critical surface mass density:

$$\Sigma_c$$

Therefore we demostrate the condition to multiple image formation from the source is: $\Sigma\rho \geq \Sigma_c$.

Obiously before all is valid only in the case the observer o, the deflector D and the source s are perfectly co-aligned.

Einstein Ring Equation:

If source, deflector or lens and observer are colinear then a special situation arise. We will have an Einsten Ring, because of there is circular symmetry. In that case we most combinated this equations, and we can obtain the Einstein radius:

13 ⟶ $\beta_E = \frac{4GM}{c^2 R}$

14 ⟶ $\varphi = \frac{\rho}{p}$

15 ⟶ $\frac{\varphi_0}{q} \approx \frac{\beta_0}{q + p}$

and we can obtain the Einstein radius, (Einstein Ring):

16 ⟶ $\varphi_E = \left[\frac{q}{p(q + p)} \frac{4GM}{c^2} \right]^{\frac{1}{2}}$

We have listed in table 1some stellar objects to satisfied multiple images condition, we can considered **q=p**, an take value for s to satisfied the multiple images condition.

For instance of Gravitational Lenses (GL)

Parameter	Neutron	Earth	Jupiter	Sun	Star	Galaxy	Measure Unit
M	$1.7\text{x}10^{-27}$	$6\text{x}10^{24}$	$1.9\text{x}10^{27}$	$2\text{x}10^{30}$	$3\text{x}10^{30}$	$1\text{x}10^{43}$	kg
R	$1\text{x}10^{-15}$	$6.4\text{x}10^{6}$	$7\text{x}10^{7}$	$7\text{x}10^{8}$	$1\text{x}10^{4}$	$3\text{x}10^{20}$	m
s	$2\text{x}10^{25}$	$2\text{x}10^{16}$	$1\text{x}10^{16}$	$1\text{x}10^{15}$	$1\text{x}10^{9}$	$1\text{x}10^{32}$	m
φ_E	$3.5\text{x}10^{-40}$	$6.6\text{x}10^{-10}$	$1.6\text{x}10^{-8}$	$1.6\text{x}10^{-6}$	$2\text{x}10^{-3}$	$1.2\text{x}10^{-8}$	arcsec
$\sum\rho \big/ \sum_c$	1.2	1.06	1.1	1.15	1.1	1.4	
$\sum R \big/ \sum_c$	$5\text{x}10^{1}$	4.4	5.7	6.1	$4\text{x}10^{4}$	$1.6\text{x}10^{7}$	
F(ρ)	$8\text{x}10^{24}$	$9\text{x}10^{15}$	$4.5\text{x}10^{15}$	$4.2\text{x}10^{14}$	$4.5\text{x}10^{8}$	$3.4\text{x}10^{31}$	m
F(R)	$2\text{x}10^{23}$	$2.2\text{x}10^{15}$	$8.8\text{x}10^{14}$	$8.3\text{x}10^{13}$	$1\text{x}10^{4}$	$3\text{x}10^{24}$	m
ρ	$7\text{x}10^{-15}$	$1.3\text{x}10^{7}$	$1.6\text{x}10^{8}$	$1.6\text{x}10^{9}$	$2\text{x}10^{6}$	$1\text{x}10^{24}$	m

Dates Table 1

In conclusion:

1. When we calculated the deviation angles on the classical mechanics and relativistic frames, we finded $\beta_E = 2\beta_N$
2. To compare the gravitational and optical lenses, we conclude the analogy of this compare and find one peculiarity: **the focal distance in the optic thin lens is constant, in the gravitational lenses the focal distance depend of impact parameter ρ .**
3. We find solution in teo cases:
   a) When the source s, deflector D and observer o are alignment perfect.
   b) When thw deflector D, observer o and source s are not alignment.
4. We obtained the condition for multiple images formation produced for a spheric geometry mass distribution.
5. We finded the equation for Eisntein Ring.

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