Shear effects in microlensing of large sources

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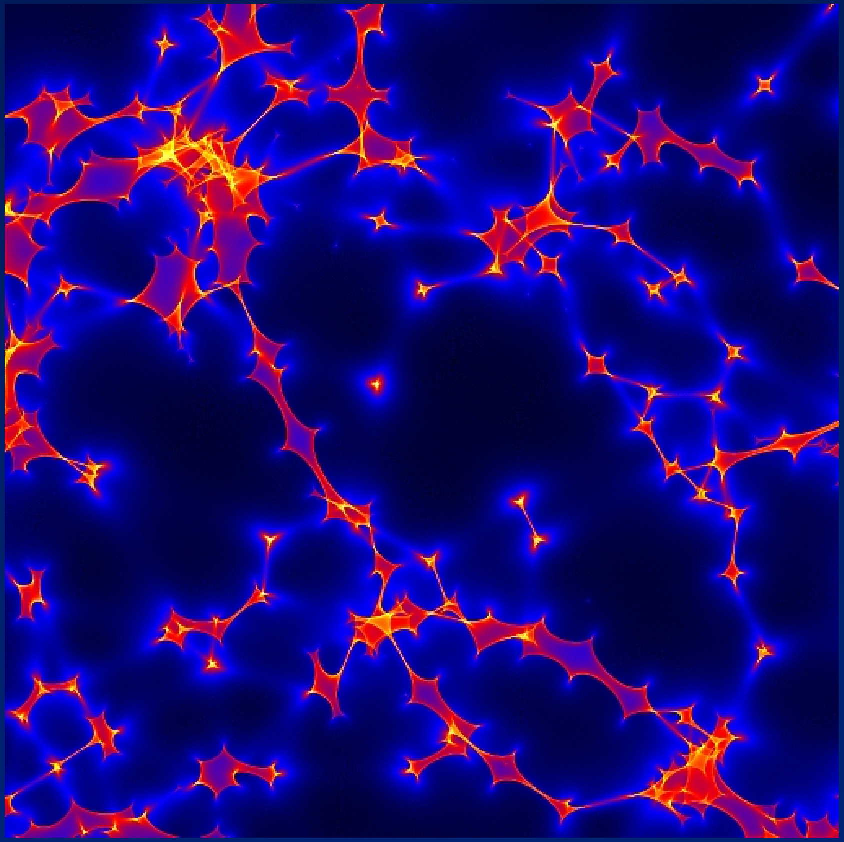
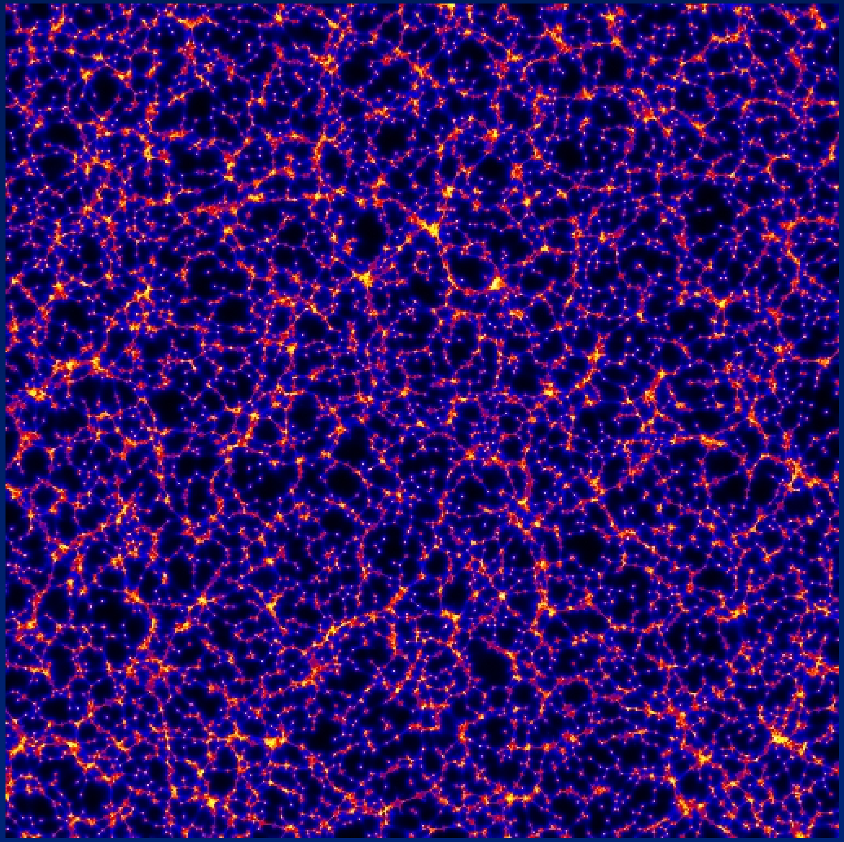
Shear effects in microlensing of large sources

- Why analytical microlensing of large sources?
- The work of Refsdal & Stabell ($\gamma = 0$)
- R&S with shear
- New calculation of variation
  - ⋆ Effect of a single lens
  - ⋆ Statistical effect of many lenses
- Comparison with numerical simulations
- Summary

Why analytical microlensing of large sources?

$32\,R_\mathrm{E}$ $\qquad$ $\kappa = 0.3,\ \gamma = 0$ $\qquad$ $256\,R_\mathrm{E}$





- complex magnification map
- more regular on large scales

Previous analytical work

- *Deguchi & Watson (1987), PRL 59, 2814*
  - ⋆ semianalytic calculation of fluctuation variance
- *Seitz & Schneider (1994), A&A 288, 1*
  Seitz et al. (1994), A&A 288, 19
  - ⋆ extend to autocorrelation function
  - ⋆ demanding numerics
- *Neindorf (2003), A&A 404, 83*
  - ⋆ simplified integrals
  - ⋆ numerical solution

⇝ analytical microlensing of arbitrary sources difficult

Microlensing of large sources

- magnification map uncorrelated on scales $\gg R_{\mathrm{E}}$
- Total noise depending on number N of fluctuation cells

$$N \propto \left(\frac{R_{\mathrm{S}}}{R_{\mathrm{E}}}\right)^2$$

$$\text{fluctuation} \propto \sqrt{N} \propto R_{\mathrm{S}} \qquad \text{mean} \propto R_{\mathrm{S}}^2$$

$$\frac{\delta\mu}{\langle\mu\rangle} \propto R_{\mathrm{S}}^{-1}$$

- quantified by *Refsdal & Stabell (1991), A&A 250, 62* (without shear)

The R&S approach for large sources (no shear)

[Refsdal & Stabell (1991), A&A 250, 62]

- effect of lenses equivalent to smooth matter distribution
- magnification depends on mean matter density κ
- number of stars in front of projected source

$$\langle N \rangle = |\mu| \, \kappa \left(\frac{R_\mathrm{S}}{R_\mathrm{E}} \right)^2$$

- Poisson scatter with mean $\langle N \rangle$: rms $\delta N = \sqrt{\langle N \rangle}$
- scatter in N translated to scatter in mean density κ

$$\frac{\delta \kappa}{\kappa} = \frac{\delta N}{\langle N \rangle} = \frac{1}{\sqrt{\langle N \rangle}}$$

R&S magnification variations

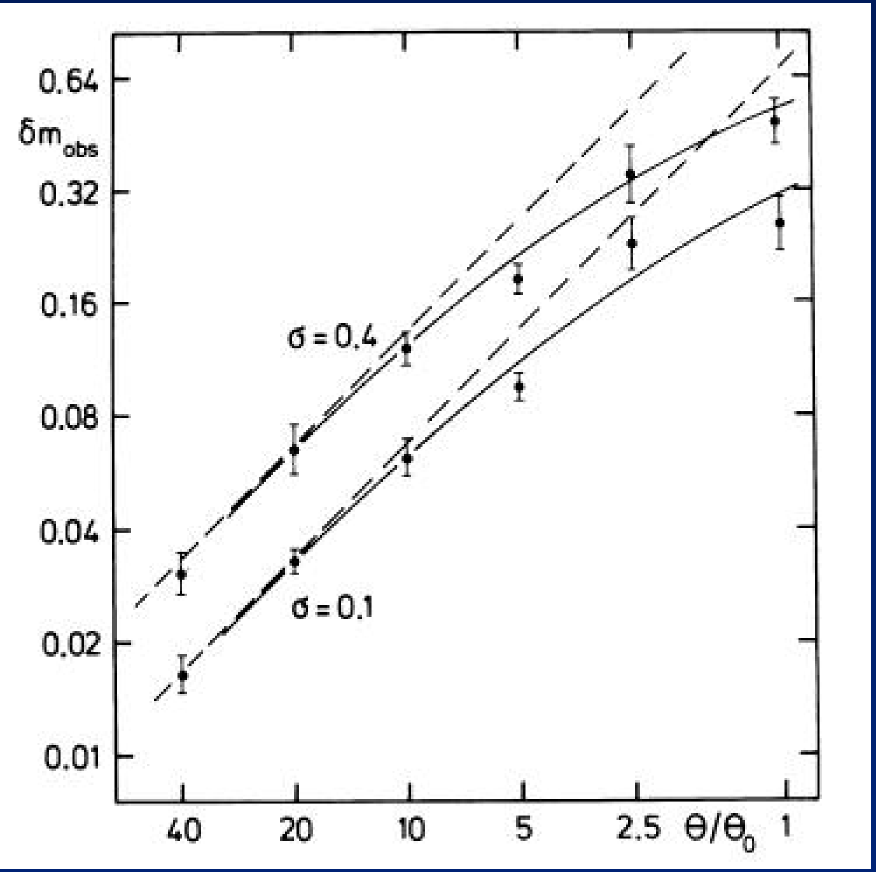
- mean number of stars $$\langle N \rangle = |\mu|\,\kappa \left(\frac{R_\mathrm{S}}{R_\mathrm{E}} \right)^2$$
- density scatter $$\frac{\delta\kappa}{\kappa} = \frac{1}{\sqrt{\langle N \rangle}}$$
- magnification $$\mu = (1-\kappa)^{-2}$$
- combine equations

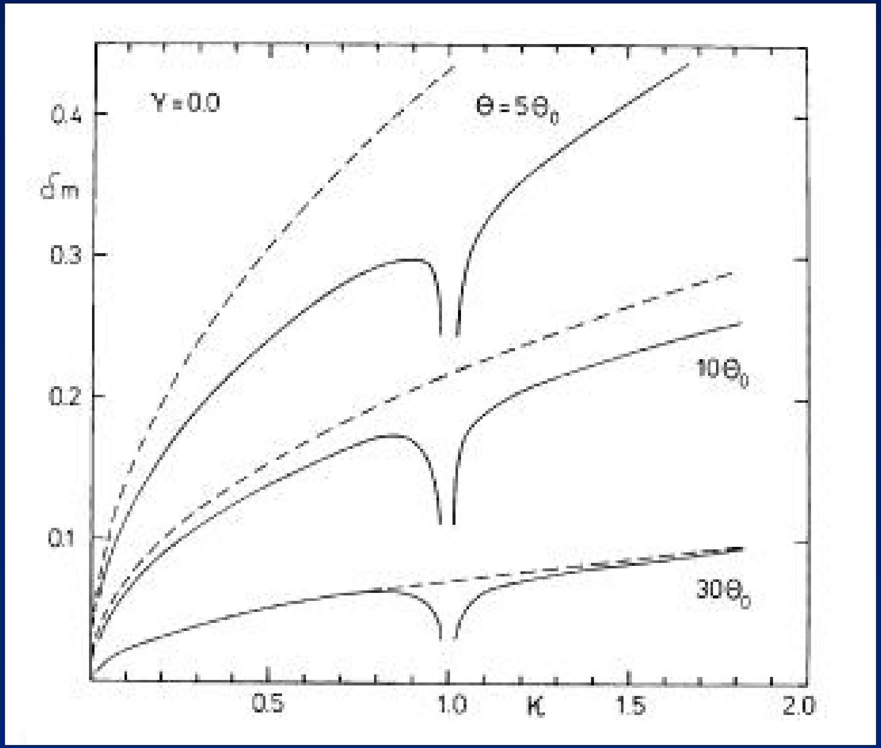
$$\frac{\delta\mu}{\mu} = \frac{1}{\mu}\frac{\partial\mu}{\partial\kappa}\delta\kappa = 2\sqrt{\kappa}\frac{R_\mathrm{E}}{R_\mathrm{S}}$$

- confirms earlier result of $\delta\mu/\mu \propto R_\mathrm{S}^{-1}$

Comparison with simulations (without shear)

[Refsdal & Stabell (1991), A&A 250, 62]





[Refsdal & Stabell (1997), A&A 325, 877]

R&S with shear?

- magnification $$\mu^{-1} = (1-\kappa)^2 - |\gamma|^2$$
- combine equations

$$\frac{\delta\mu}{\mu} = 2(1-\kappa)\sqrt{|\mu|\kappa}\,\frac{R_{\mathsf{E}}}{R_{\mathsf{S}}}$$

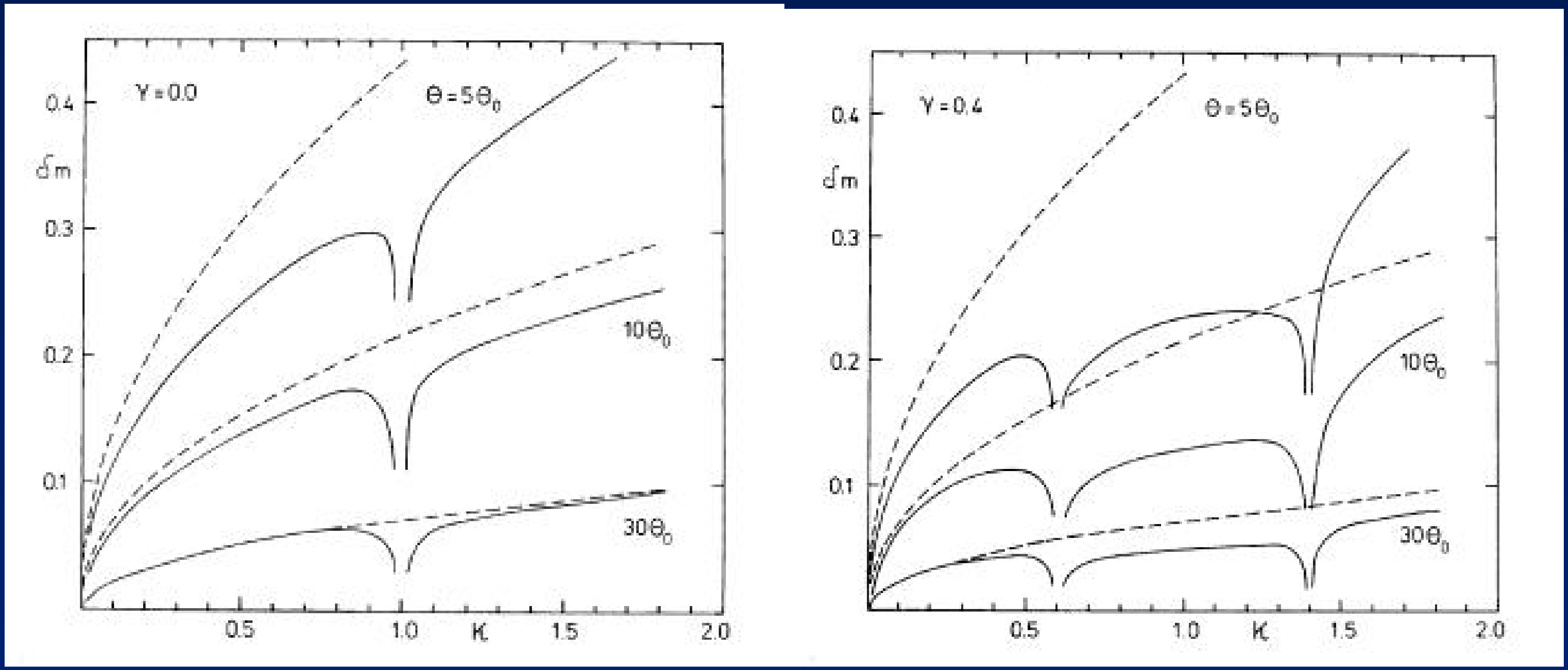
$$\left(\frac{\delta\mu}{\mu}\right) \Big/ \left(\frac{\delta\mu}{\mu}\right)_{\gamma=0} = (1-\kappa)\sqrt{|\mu|}$$

$$= \left|1 - \frac{|\gamma|^2}{(1-\kappa)^2}\right|^{-1/2}$$

- for $|\gamma| < 2\,|1-\kappa|$: correction factor > 1

not correct!

Comparison with simulations (with shear)



[Refsdal & Stabell (1997), A&A 325, 877]

Fluctuation with shear always smaller than without!

Why does it not work with shear?

- magnification $\mu^{-1} = (1-\kappa)^2 - |\gamma|^2$ only for circular disc
  - ⋆ image of circular source is elliptical for $\gamma \neq 0$
  - ⋆ but: works for elliptical sources and $\gamma = 0$

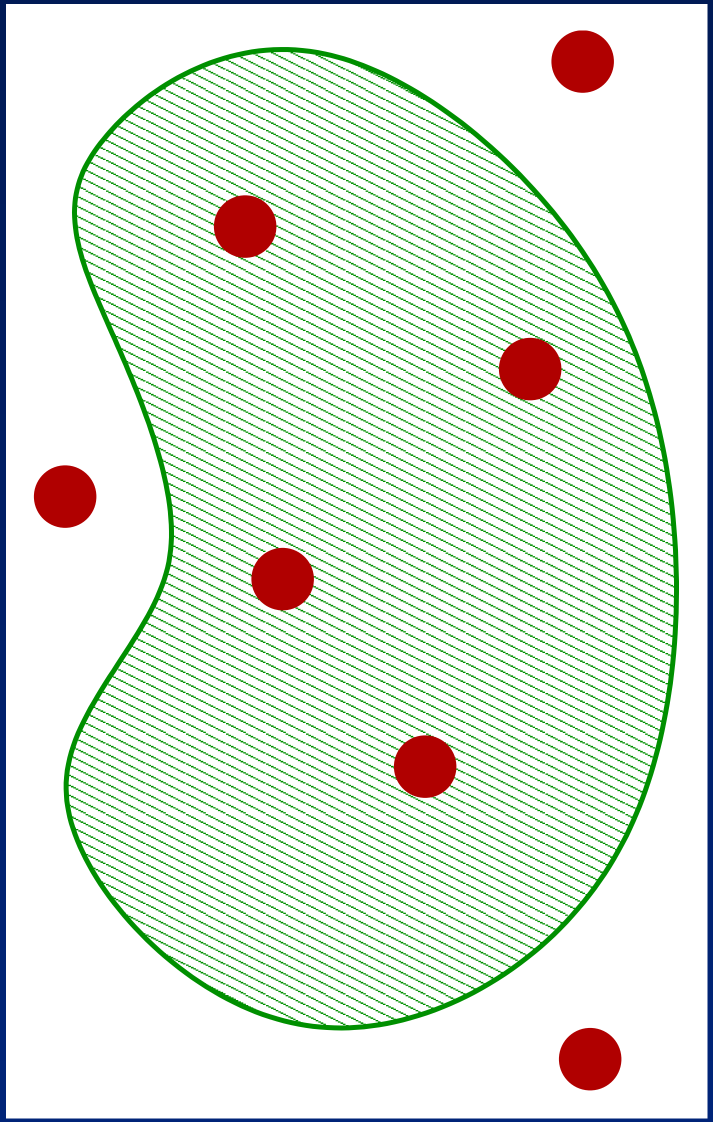
- Poisson fluctuations will also change γ
  - ⋆ lenses outside of source area will contribute!

Better questions:

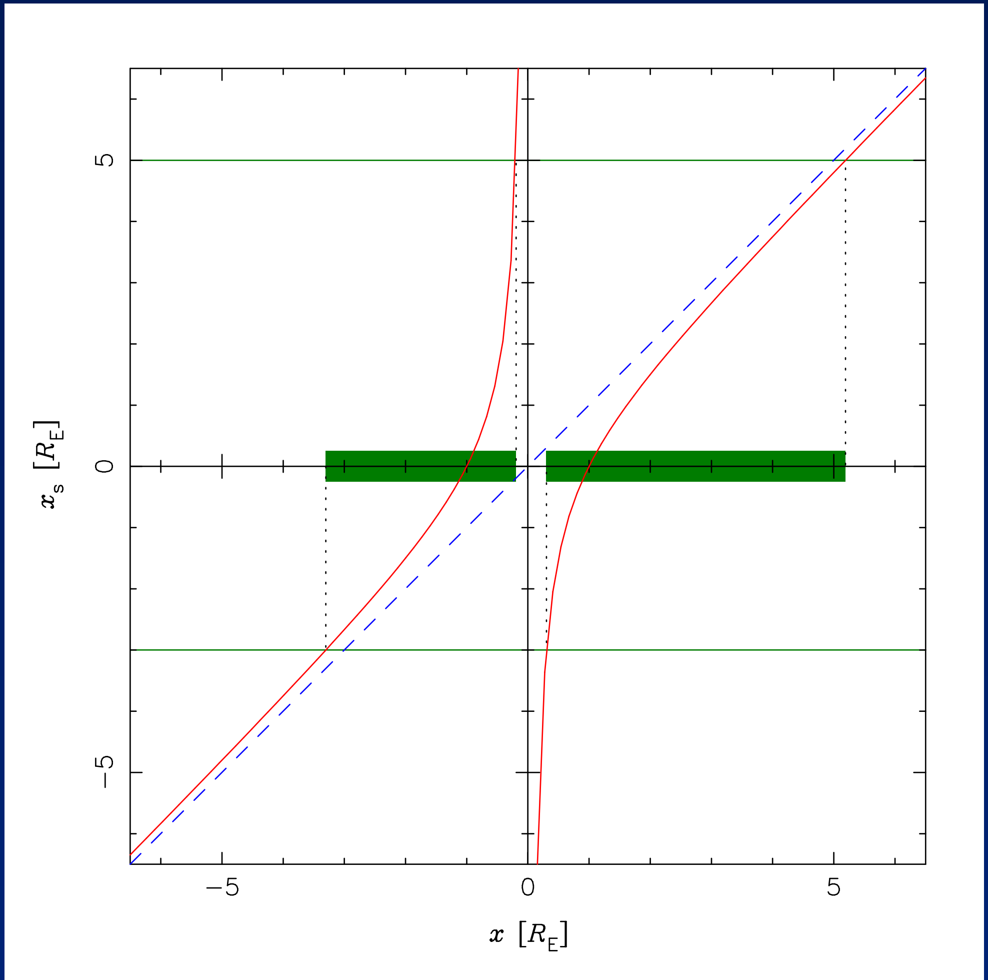
- Why does it work without shear?
- Can we replace collection of point mass lenses by smooth mean density?

View of the source plane

- asymptotic additional magnification far away from a point lens ($r \propto r_\mathrm{S} \gg R_\mathrm{E}$)
  - ⋆ no shear $\Delta\mu \sim (r_\mathrm{S}/R_\mathrm{E})^{-4}$
    - ∗ effect confined to a few R_E
    - ∗ constant inner and no outer contribution
  - ⋆ with shear
    $$\Delta\mu \sim \gamma(r_\mathrm{S}/R_\mathrm{E})^{-2} + (r_\mathrm{S}/R_\mathrm{E})^{-4}$$
    - ∗ effect at large distances
    - ∗ possibly varying inner and non-vanishing outer contribution



Concept of single-lens calculation (1)



- lens produces hole of radius $\sim R_E^2/R_S$

  $\rightsquigarrow$ $\Delta F \sim \pi R_E^4/R_S^2$ (insignificant)

- lens shifts outer limb by $\sim R_E^2/R_S$

  $\rightsquigarrow$ $\Delta F \sim 2\pi R_E^2$ (leading term)

$\rightsquigarrow$ relative effect: $\Delta F/F \sim R_E^2/R_S^2$ (small)

Concept of single-lens calculation (2)

- use magnification for amplification
- calculate areas by using line integrals along boundaries

$$F = \frac{1}{2} \oint \mathrm{d}\phi \, r^2(\phi)$$

- complex formalism with $z = r\mathrm{e}^{i\phi}$

$$F = \frac{1}{2\mathrm{i}} \oint \mathrm{d}z \, \overline{z}$$

$$F_{\mathsf{S}} = \frac{1}{2\mathrm{i}} \oint \mathrm{d}z_{\mathsf{S}} \, \overline{z}_{\mathsf{S}}$$

Parametrization of sources/images

- elliptical source/images
- major and minor axes A and B of *image*
- position angle θ
- complex parameters

$$\mathfrak{a} = \frac{A+B}{2} \qquad \mathfrak{b} = \frac{A-B}{2}\,\mathrm{e}^{2\mathrm{i}\theta}$$

- boundary of projected source (relative to lens)

$$z = \mathfrak{a}\mathbf{u} + \mathfrak{b}\overline{\mathbf{u}} - z_0 \qquad |\mathbf{u}| = 1$$

The single lens integral (1)

- start with $$z = \mathbf{a}\mathbf{u} + \mathbf{b}\overline{\mathbf{u}} - z_0$$
- image plane $$F = \pi(|\mathbf{a}|^2 - |\mathbf{b}|^2) = \pi AB$$
- source plane: apply lens equation

$$z_s = (1-\kappa)\,z + \gamma\,\overline{z} - \frac{R_E^2}{\overline{z}}$$

$$F_S = \frac{1}{2i}\oint \mathrm{d}z_S\,\overline{z}_S$$
$$= \frac{1}{2i}\oint \left[(1-\kappa)\mathrm{d}z + \gamma\mathrm{d}\overline{z} + \frac{R_E^2}{\overline{z}^2}\mathrm{d}\overline{z}\right]\left[(1-\kappa)\overline{z} + \overline{\gamma}z - \frac{R_E^2}{z}\right]$$

The single lens integral (2)

- expand up to order $R_\mathsf{E}^2/R_\mathsf{S}^0$, neglecting $R_\mathsf{E}^4/R_\mathsf{S}^2$ etc.
- most terms trivial
- some simple Cauchy integrals
- one non-trivial integral left:

$$F_\mathsf{S} = \frac{F}{\mu} - 2\pi R_\mathsf{E}^2 (1-\kappa) \cdot \begin{cases} 1 & \text{lens inside} \\ 0 & \text{lens outside} \end{cases}$$

$$- R_\mathsf{E}^2 \,\mathrm{Im}\left(\gamma \oint \frac{\mathrm{d}\overline{z}}{z} \right)$$

- last term depends on shape of source!

Additional relative magnification

. . . skipping the integration . . .

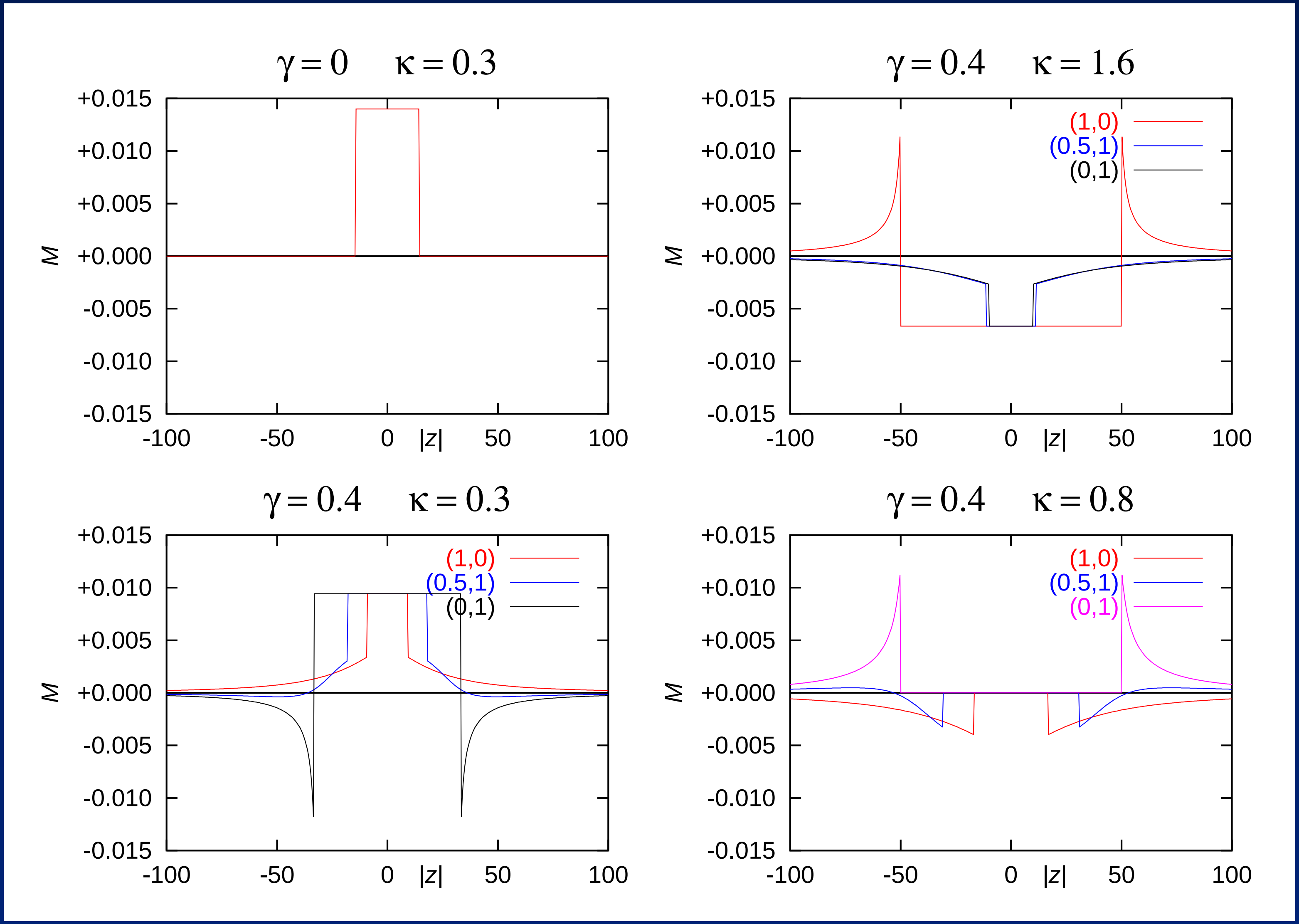
$$M := \frac{F}{\mu F_\mathsf{S}} - 1$$

$$= \begin{cases} \dfrac{2\pi R_\mathsf{E}^2}{F_\mathsf{S}} \left[(1-\kappa) + \mathrm{Re}\left(\gamma \dfrac{\overline{\mathfrak{b}}}{\mathfrak{a}} \right) \right] & \text{inside} \\ 2\mu R_\mathsf{E}^2 \,\mathrm{Re}\left\{ \dfrac{\gamma}{2\mathfrak{a}\mathfrak{b}} \left[\left(1 - \dfrac{4\mathfrak{a}\mathfrak{b}}{z_0^2} \right)^{-1/2} - 1 \right] \right\} & \text{outside} \end{cases}$$

- circular source: $\mathfrak{a} = (1-\kappa)\mu R_\mathsf{S}$ $\mathfrak{b} = -\gamma \mu R_\mathsf{S}$
  (vice versa for $\mu < 0$)

Outside contribution does not vanish!

Additional relative magnification: plots



$$(\; R_{\mathsf{S}} = 10 \qquad R_{\mathsf{E}} = 1 \;)$$

Statistical ensemble of lenses

- describe mean density of lenses as constant κ
- constant γ
- calculate fluctuations statistically
- assume small relative fluctuations
  - ⋆ generally valid for large sources

$\rightsquigarrow$ effects of individual lenses add up linearly

Integrated Poisson statistics

- consider small part of lens plane F
- number of lenses $\langle N \rangle = \kappa \dfrac{F}{\pi R_{\mathsf{E}}^2}$
- variance $(\delta N)^2 = \langle N \rangle$
- variance of additional relative magnification

$$\left(\frac{\delta\mu}{\mu}\right)^2 = (\delta N)^2 M^2$$

- several small parts: variances additive

$$\left(\frac{\delta\mu}{\mu}\right)^2 = \frac{\kappa}{\pi R_{\mathsf{E}}^2} \iint \mathrm{d}^2 z_0 \, M^2(z_0)$$

Total fluctuations

$$\left(\frac{\delta\mu}{\mu}\right)^2_{\text{tot}} = \left(\frac{\delta\mu}{\mu}\right)^2_{\text{inner}} + \left(\frac{\delta\mu}{\mu}\right)^2_{\text{outer}}$$

$$= \underbrace{\frac{4\kappa}{F_{\mathsf{S}}/(\pi R_{\mathsf{E}}^2)}}_{\text{R\&S }(\gamma=0)} \quad \mu \cdot$$

$$\cdot \left\{ \underbrace{\left[1-\kappa+\mathrm{Re}\left(\gamma\frac{\overline{\mathbf{b}}}{\mathbf{a}}\right)\right]^2}_{\text{inner}} + \underbrace{\frac{|\gamma|^2}{2}\left(1-\frac{|\mathbf{b}|^2}{|\mathbf{a}|^2}\right)}_{\text{outer}} \right\}$$

Total fluctuations for circular sources

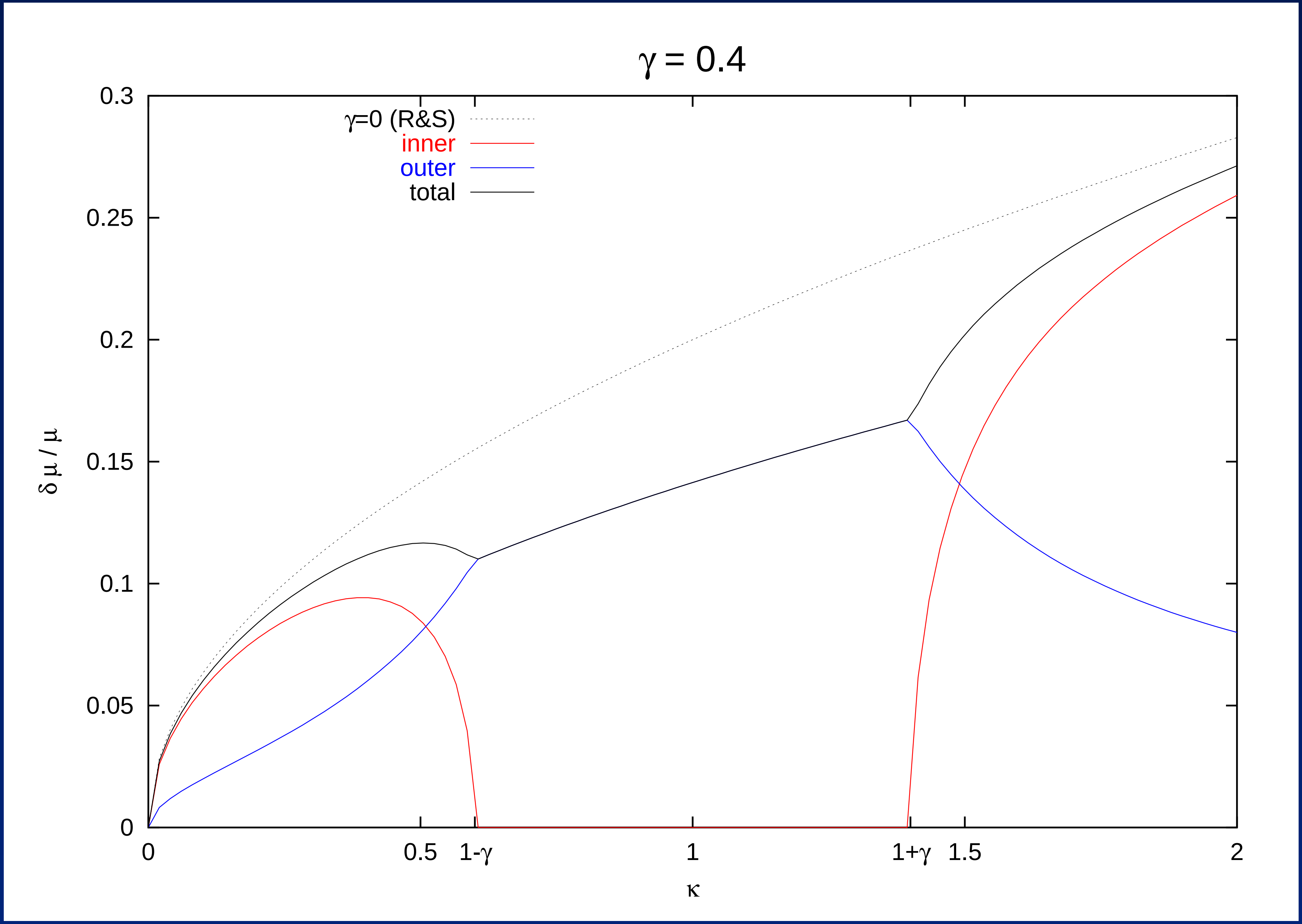
$$\left(\frac{\delta\mu}{\mu}\right)^2_{\text{tot}} = \left(\left.\frac{\delta\mu}{\mu}\right|_{\gamma=0}\right)^2 \cdot \begin{cases} 1 - \frac{1}{2}\frac{|\gamma|^2}{(1-\kappa)^2} & \text{for } \mu > 0 \\ \frac{1}{2} & \text{for } \mu < 0 \end{cases}$$

- $\mu > 0$: inner and outer contribution

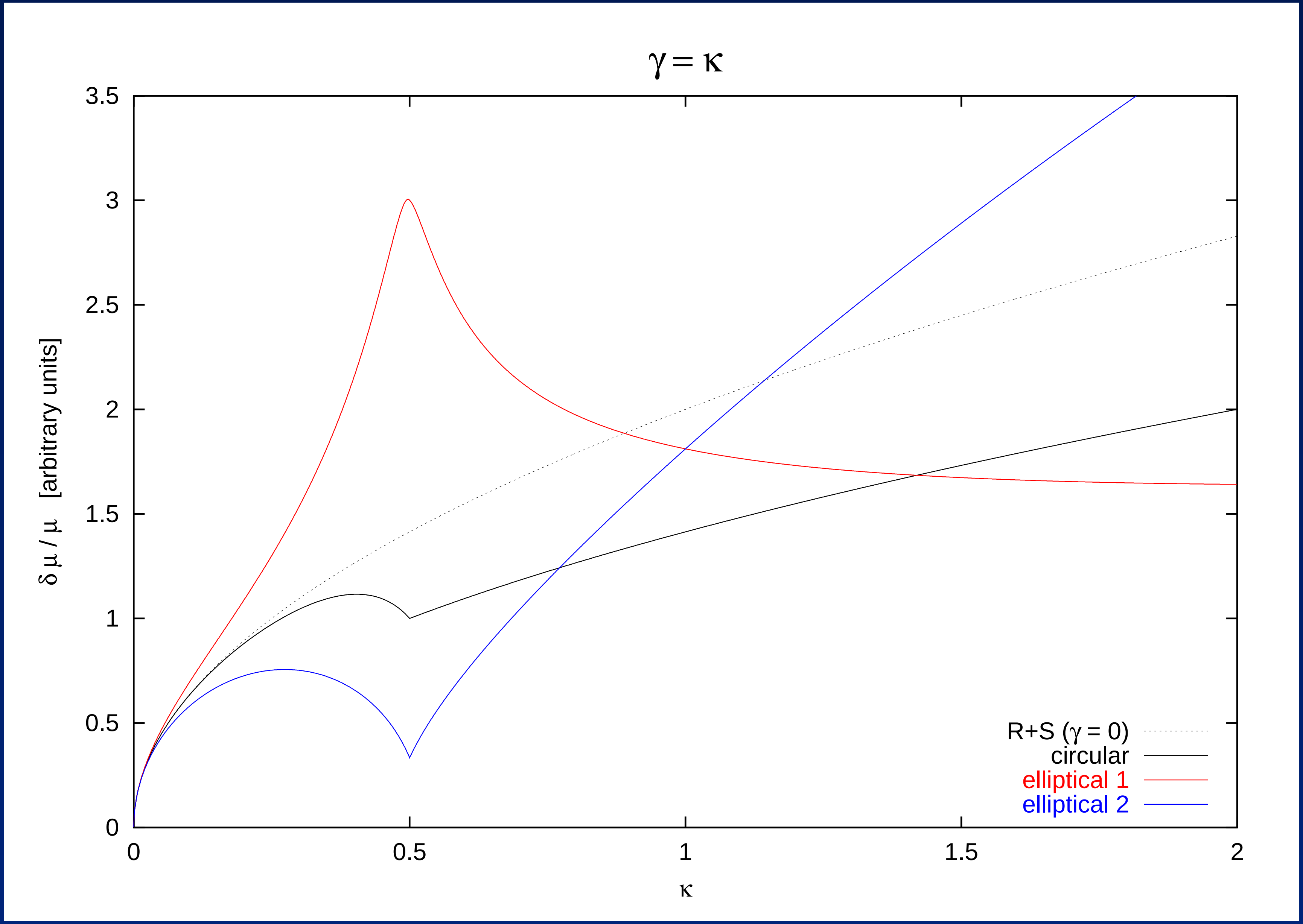
$$1 - \frac{|\gamma|^2}{(1-\kappa)^2} \quad \text{and} \quad \frac{1}{2}\frac{|\gamma|^2}{(1-\kappa)^2}$$

- $\mu < 0$: only outer contributions

Total fluctuations for circular sources: plots



Total fluctuations for elliptical sources: plots

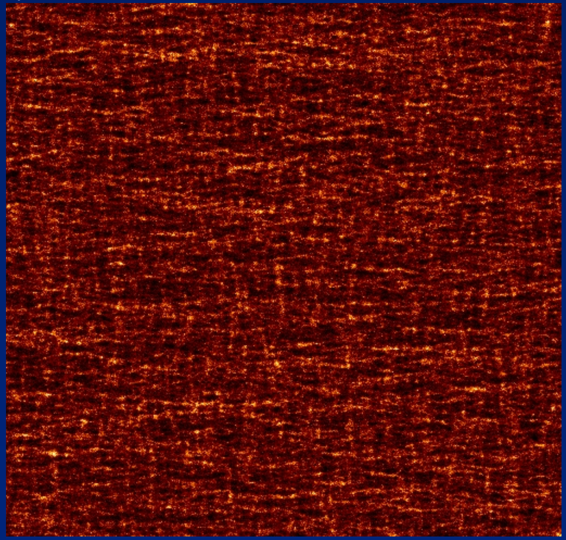


Numerical simulations

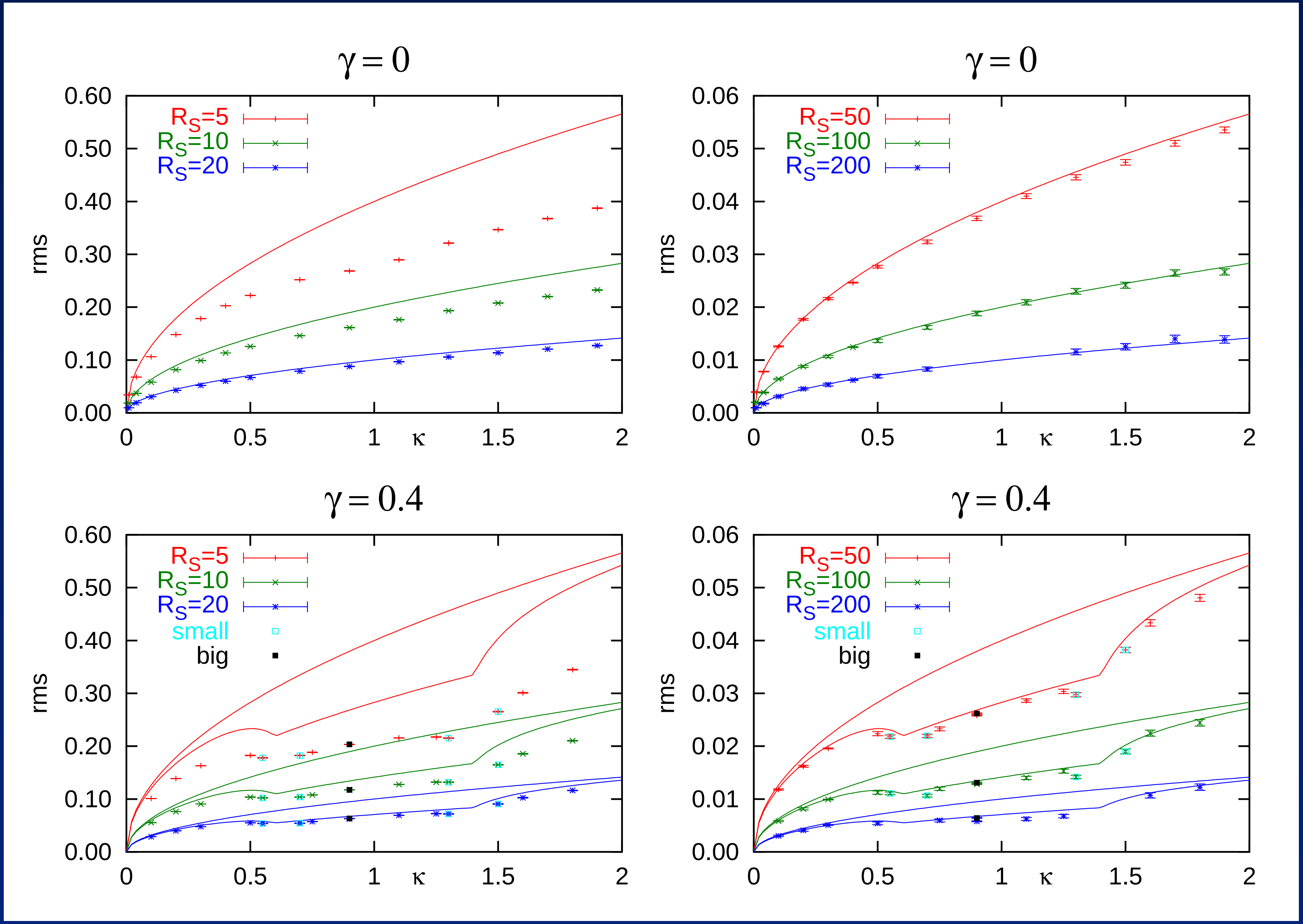
- ray shooting treecode of Joe Wambsganß
- large ray shooting fields: $\sim 500\, R_{\mathrm{E}}$
- larger star fields: $\sim 1000\, R_{\mathrm{E}}$
- large pixels: $\sim 1\, R_{\mathrm{E}}$
- low ray density
- hundreds of independent realisations
- convolution with circular source
- variation statistics

$\gamma = 0.40 \qquad \kappa = 0.50$

$\gamma = 0.40 \qquad \kappa = 0.90$



Numerical simulations: results



Summary

- Work of Refsdal & Stabell generalized for external shear
- Magnification by single lens
  - ⋆ constant but shape dependent inside of source area
  - ⋆ significant effect outside of source area
  - ⇝ correlation on scales $> R_S$
- Variations calculated as integrated Poisson scatter
- Result confirmed by simulations
- Outlook
  - ⋆ autocorrelation function
  - ⋆ relation to saddle point micro-/millilensing
  - ⋆ non-circular sources

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